

Generalization of Visual Pattern

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Abstract: *The study describes the process of visualisation of patterns generalization. Visual pattern is the pattern expressed in image form. The subject of this research is one male student of 7th grade of the Junior High Schools in Tuban who successfully generalizes the pattern correctly and used symbolic generalization. Data collect by pattern generalization tasks (PGT) and interviews. In the first stage, the students completed the task by think aloud and in the second stage the researchers conducted a task-based interview to understand the generalization process of the pattern undertaken by the chosen subject. The results show that the process of generalizing patterns on successful subjects generalizes the pattern correctly using the following steps; (1) Discovering of regularity, (2) confirming of regularity, (3) Generalizing the n-th image and (4) proving the truth of the formula.*

Keywords – Generatization, Visual Pattern

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I. Introduction

Patterns are important topics, underlie learning and mathematical thinking. Some mathematicians argue that mathematics is "the science of pattern" [1] [2]. This perspective highlights the existence of patterns in all areas of mathematics. In particular, some researchers seen pattern as a way of developing algebraic thinking and fundamental step to build generalizations. Pattern is the essence of mathematics [3].

Generalization is an important aspect in all areas of mathematics and is a competence in learning mathematics at all levels. For example, in the arithmetic, the student can generalize that the multiplication of integer by the number 5 will produce integers with the last digits 0 or 5. In the geometry, the theorem can be regarded as the product of generalization [4]. In the algebra, [5] emphasize that variables are common tools used to represent generalizations. The process of pattern generalization is interesting to study, because pattern generalization can contribute to the development of problem-solving skills, through the emphasis of analyzing certain cases, organizing data systematically, conjecturing and generalizing. Working with numerical, geometric and pictorial patterns can assist in building mathematical significance and contributing to the development of some skills related to problem solving and algebraic thinking [6]. In addition, generalization of geometric patterns can develop students' visualization, reasoning and argumentation skills [7] [8].

Curriculum 2013 In Indonesia emphasize students at junior high school competen in identifying patterns and using them to predict general rules and provide predictions [9]. Similarly, in the document of National Council of Teachers of Mathematics (NCTM)[10], one of the algebraic standards that should be studied by students in 6th grade through 8th grade is understanding of patterns. In understanding of pattern students are required to represent, analyze and generalize variations of patterns with tables, graphs, words, and symbols.

Generalization process is often done to analyze the pattern. [11] use three models of pattern analysis and pattern relationships based on the Smith model framework in discussing functional thinking, namely (1) recursive patterning which includes the discovery of variation in a sequence of values, (2) covariational thinking based on analysis two variations of simultan quantity and understanding that change is something explicit and the dynamic part of the function description and (3) correspondence relationship is based on the correlation between the variables.

A significant contribution to the generalization process has been given by some researchers [12], [13], [14], [15]. [12] states four steps of algebraic generalization of patterns, (1) particularisation, (2) noting similarities, (3) hypothesizing, and (4) generating expression for nth image. [13] identifies four stages in generalization, namely (1) specialization, (2) making conjectures, (3) symbolizations, and (4) generalizations. And then [14]) states four stages to describe the process of pattern generalization, namely (1) direct modeling stage, (2) pattern identification stage, (3) stage proving pattern testing, and (4) stage finding rules for general case. Furthermore, [15] use three steps to produce generalizations, namely; (1) discovery of a regularity, (2) confirming of regularity and (3) generalization and conclusion. The research will explore the process of pattern generalization of 7th Grade Junior High School who have succeeded in making the general formula of Pattern Generalization Task (PGT).

II. Method

This research includes qualitative research with descriptive explorative approach. The subject of this research is a 7th Grade Junior High School student of who successfully generalize the pattern correctly and produce symbolic generalization. The selection of subject is based on the result of doing of pattern generalization task (PGT) by 7th Grade Junior High School student in a junior high school in Tuban, East Java, Indonesia. Instruments used in this study there are two types of main instruments and auxiliary instruments. The main instrument is the researcher himself who acts as planner, data collector, data analyzer, data interpreter, and research reporter. Auxiliary instruments used in this research are pattern generalization task (PGT) and task-based interviews. PGT is used to obtain a picture of the thinking of pattern generalization process. An interview used is an unstructured interview. From the results of PGT and interview work, then the data will be analyzed include the stages; transcoding the obtained data, reducing data, generating data codes, describing the process of generalizing patterns and drawing conclusions. The PGT problem is as follows:

Look at the following diagrams containing squares and triangles built from matchsticks.

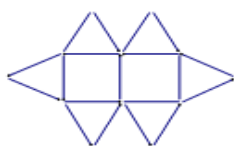
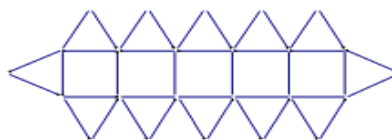


Diagram a



- How many matchsticks will you need in total if there are 6 squares?
Explain and/or show how you got to your answer.
- How many matchsticks will you need in total if there are 10 squares?
Explain and/or show how you got to your answer.
- How many matchsticks will you need in total if there are 50 squares?
Explain and/or show how you got to your answer.
- Give an algebraic “rule” or “formula” to work out the total number of matchsticks you will need if there are “n” squares

III. Result and Discussion

3.1 Result

The research begins by giving a pattern generalization task (PGT) on the student of 7th Grade Junior High School student in Tuban, East Java, Indonesia which amounted to 29 students. From the 29 students who have been working on PGT, only six students have successfully generalized the pattern correctly and use symbolic generalization. From the six students, one student selected to be examined more deeply, namely SK. Furthermore, researchers conducted a job-based interview on the SK to clarify the written answer from PGT as well as to know the SK’s process of thinking in pattern generalization.

When given PGT, SK observes Diagram a and Diagram b alternately. SK observation is focused on triangle shape on each diagram. SK observed the upper triangle, then SK observed the lower triangle and continued with right triangle and left side triangle. SK multiplies the number of triangles by 3. This is because 1 triangle is made up of 3 matchsticks. SK then counts the matchsticks that have not been counted. Once convinced that all matchsticks have been identified and counted, SK next SK sums up the results of matchstick calculations that make up the triangle and the result of calculating the matchsticks in the middle.

SK uses the same strategy as Diagram a in identifying the number of matchsticks that make up Diagram b. According to him, this is because in Diagram b the arrangement of matchstick is the same as Diagram a. SK has found the regularity of Diagram a and Diagram b. The SK visualization process in identifying the regularity of Diagram a and Diagram b can be seen in Figure 1 below.

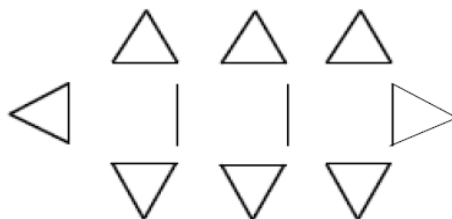


Figure 1. SK’s visualization process in identifying the regularity of Diagram a and Diagram b

In solving the question a, SK draws a matchstick configuration consisting of 6 squares like diagram a and diagram b. This picture really helps SK in answer questions a. SK focuses its attention on the triangle shape, then uses the same way as used for diagram a and diagram b. SK has confirmed the regularity. The visualization of SK in answering question a can be seen in Figure 2 below.

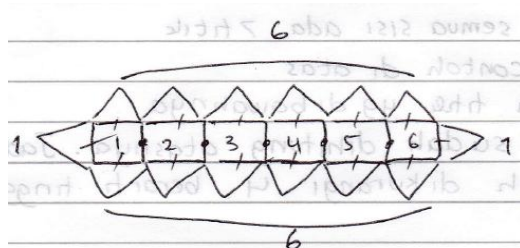


Figure 2. SK's visualization in answering question a

Similarly to question a, in solving the question b, SK draws a matchstick configuration consisting of 10 squares like diagram a and diagram b. SK focuses his attention on the shape of the triangle. SK applies the regularity he used before. SK has confirmed regularity.

In solving the question c, SK draws a sketch of a matchstick configuration that looks as if it consists of 50 square. SK does not draw completely. SK applies the regularity he used before. SK does not change its strategy. SK has confirmed the matching rules that have been used previously. The following figure is a SK's visualization in answering questions c.

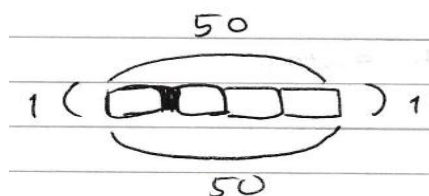


Figure 3. SK's visualization in answering question b

SK does not change its visualization strategy. He has confirmed the truth of order. Based on the visualization strategy previously used, SK produced a general formula for determining the number of matchsticks for n-square, ie $3n + 3n + 6 + (n-1)$. This formula is obtained from the visualization process as shown in Figure 4 below.

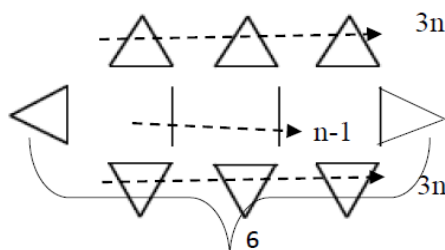


Figure 4. SK's visualization of the pattern generalization

SK tested the truth of the general formula that has been obtained by using inductive proof, that is by reconciling the answers of questions a, b, and c by replacing $n = 6$, $n = 10$, and $n = 50$. After getting the same result, SK is correct is convinced by the general formula which is the result of the generalization process.

3.2 Discussion

Based on the results of the research, the pattern generalization process of SK include four stages, namely; (1) discovering of regularity, (2) confirming of regularity, (3) generalization and (4) proving the truth of generalization. For phase (1) to stage (3) corresponds to the process generalization stage of the pattern proposed by [15]. While stage (4) is a stage that is not in the reseach of [15] but occurs in this research.

At the stage of discovery of regularity, SK Observing Figures 1 and 2 uses a visualization strategy with a global approach type I, which is by observing images with a holistic view and not overlap. Visualization with global approach does not use iteration additions. SK is able to capture the similarity or regularity of the pattern image. [12] states that the generalization of algebraic patterns rests on the ability to capture similarities and provide some explanation of the sequence.

At the stage of confirming of regularity confirms the truth of the regularity generated by a particular example. [16] argue that validating the conjecture is the determination of the truth for a particular case but not in general. That is, that before entering the stage of generating the general formula, must be tested conjecture, even if necessary repeatedly. Pattern generalization product produced by SK is a symbolic generalization, which states the result of generalization in the form of a variable. [17] that the type of symbolic generalization is a type of generalization associated with algebraic objects or symbols that are not limited to a particular object.

IV. Conclusion

SK generalizes the pattern includes four stages namely; (1) discovering of regularity, (2) confirming of regularity, 3) *Generalizing the n-th image and* (4) proving the truth of formula. 7th grade student are still few who can make a symbolic generalization and understand the meaning of variable n. Therefore the teaching of pattern generalization and understanding of the meaning of the variable n needs stimulus in 7th grade as the beginning in algebra thinking. One of the disadvantages of this study is that the subject of research consists of only one person. So that no other possibilities of product generalization can be obtained. Therefore, it can be an input for other interested researchers to examine the process of generalizing patterns related to the creativity of pattern generalization product.

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